WAVE STRUCTURE OF COUNTERFLOWING SUPERSONIC UNDEREXPANDED JETS

E. I. Sokolov and V. N. Uskov

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Results are shown of an experimental study concerning the flow structure where two opposing coaxial underexpanded jets interact. Empirical formulas are proposed for calculating the location of shock waves and of the interface, in the case of jet interaction within the zone of first "rolls."

The flow structure where opposing coaxial supersonic underexpanded jets interact has been studied to a rather limited extent. Earlier research was only concerned with jets of equal parameter values and, overall, either with a flow mode equivalent to a supersonic jet impinging on an infinitely large plate normal to it [1] or with one of the two jets perfectly efficient and discharging from a profiled nozzle [2]. Studies were also made concerning the interaction between a jet from a sonic nozzle and an unbounded supersonic stream [3, 4]. Here we will show the results of an experimental study concerning the flow structure where opposing jets interact, over a wide range of parameter values.

The object of this study was to obtain and to analyze schlieren photographs of opposing coaxial jets discharging from two separate receivers. As the active medium we used cold air discharging into a plenum. The parameters of the nozzles are given in Table 1, the range of distances between both nozzles and the range of total pressures are given in Table 2.

A typical photograph of interacting coaxial jets is shown in Fig. 1a. The interaction of opposing jets is characterized by an interface 3 (Fig. 1b) passing through the stagnation point Q on the axis, which separates the air of one jet from that of the other jet and thus constitutes a barrier which neither jet can penetrate. As in the case of interaction with a barrier, in each jet there appear centrally located density jumps 2 and 2', which intersect the floating jumps 1 and 1', producing at point T_1 reflected jumps 5 and 5' as well as tangential discontinuities 4 and 4'. The wave structure in the jets depends essentially on their inefficiencies n_1, n_2 and on the distance L between the nozzles.

The structure variations under a varying pressure P_1, with L and P_2 constant, can be described as follows. When $P_1 = p_n$, there is no discharge from nozzle I (Fig. 2a)* and the opposing jet runs into receiver I unstably (Hartmann mode) at a frequency which is a function of the receiver volume as well as of the inefficiency n_2. Under a slightly supercritical pressure the flow structure in receiver I becomes stable (Fig. 2b). Between the jets there appears an interface 3, while the central jump 2 may become convex toward the nozzle or toward the interface. The curvature of the interface is appreciable, convex toward the receiver where the pressure P is high. As pressure P_1 rises, the interface shifts toward nozzle II and its curvature decreases. It is possible now for the interface to shift from one location to another between the nozzles. On the diagram are indicated such "jumps" of the interface toward nozzle I as well is in the opposite direction, these "jumps" being due to an increase in P_1. A further increase in P_1 results in flow instability, the mode of which may be different in each jet. Usually, the wave structure of both jets breaks down (Fig. 2c). When the inefficiencies of both jets differ by an order of magnitude, then the wave

TABLE 1. Nozzle Parameters

Nozzle No.	M	θ°	d, mm
I	1	—	12
II	2	5°	12
III	2	10°	15

* The unstable location of the discontinuities is indicated in Fig. 2 by wavy lines.

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TABLE 2. Variation Ranges of Geometrical Dimensions and of Gas-Dynamic Parameters

Receiver II	Receiver I					
	nozzle I		nozzle II		nozzle III	
—	$\frac{L}{d_I}$	P_I, atm total	$\frac{L}{p_I}$	P_I, atm total	$\frac{L}{d_I}$	P_I, atm total
Nozzle I P_{II} = 10 atm total	2, 3, 4, 9, 10	4--80	2, 3, 4, 5, 6, 8	6--80	2, 6, 10	6—70
Nozzle I P_{II} = 20 atm total	5	10—60	2, 3, 5, 6, 8	15--70	2, 6, 10	6--70
Nozzle I P_{II} = 30 atm total	—	—	3, 10	6--70	—	—
Nozzle II P_{II} = 10 atm total	3, 4, 5, 6, 7	6—80				
Nozzle II P_{II} = 20 atm total	7	30—80				

structure of the jet with the higher inefficiency may remain stable while that of the other jet breaks down. It is to be noted that, when a jet acts on an infinitely large barrier, the instability vanishes at a higher jet inefficiency [5]. Apparently, both phenomena are of the same nature.

As P_1 increases still further, the flow in both jets becomes stable (Fig. 1) and the interface shifts toward nozzle II smoothly. When the maximum diameter of one jet is much larger than that of the other, then the central jump 2 in the larger jet may become convex toward its nozzle (Fig. 2d). The intersection of such a density jump with the floating jump 1 at point T_1 is accompanied by a generation of a set of four shock waves. This case is analogous to a jet with a high inefficiency impinging on a barrier with finite dimensions. This qualitatively described pattern of interaction between opposing jets varies little while the distance L between the nozzles, the Mach number M_i in the nozzles, or the orifice diameter d_i of the nozzles change over the test range.

The purpose of this experiment was not only to study the qualitative pattern but also to obtain empirical relations by which the location of the interface and of the central jumps along the jet axes within the zone of first "rolls" could be determined. For the latter purpose, we measured the following geometrical dimensions on the schlieren photographs:

a) the distances c_1 and c_2, along the respective axis, from the nozzle exit section to the central jump in the jet;

b) the distance s_2, along the axis, from the exit section of nozzle II to the interface between jets; the difference $L - s_2$ was the distance from nozzle I to the interface between jets.

Distances c_1 and c_2 were measured accurately within 0.25 mm and distances s_2 were measured accurately within 1-2 mm, inasmuch as part of the interface area appeared very blurred on the photographs. The measured distances are evidently functions of seven parameters (L, n_i, d_i, M_i; i = 1, 2) and for an evaluation of the test results, therefore, it is worthwhile to define a few universal flow parameters. As a proper basis we have selected, as have the authors of [5], the following group

$$b = Md\sqrt{kn}.$$

In order to analyze how the location of the central jumps depends on the distance between the nozzles, we have referred the quantities $c_1 + c_2 = \sum_i c_i$ and L to the sum $b_1 + b_2 = \sum_i b_i$. As a result, all test points cluster about a single curve (Fig. 3) which fits the equation

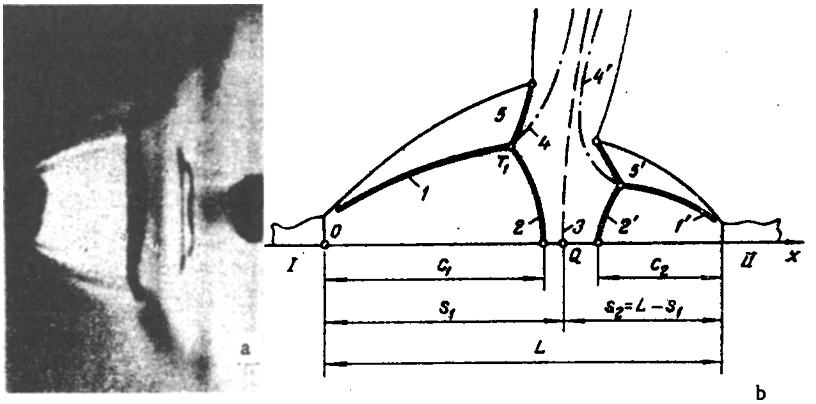

Fig. 1. (a) $M_1 = 1.0$, $n_1 = 21.8$, $d_1 = 12$ mm, $M_2 = 1.0$, $n_2 = 5.28$, $d_2 = 12$ mm, $L/d = 4$; (b) schematic diagram of the flow structure in opposing jets interacting within their first "rolls."

$$\sum_i c_i \Big/ \sum_i b_i = 0.745 - 0.83\exp\left(-1.73\frac{L}{\sum_i b_i}\right). \qquad (1)$$

The coefficients in Eq. (1) have been calculated by the method of least squares, with a 0.024 dispersion of test points about this curve.

Analogously to the location of the interface, the distances s_2 and $s_1 = L - s_2$ have been evaluated in terms of

$$c_i/b_i = f(s_i/b_i); \quad i = 1;\ 2.$$

According to Fig. 4, all test points cluster about a curve which fits the equation

$$c_i/b_i = 0.745 - 0.83\exp\left(-1.73\frac{s_i}{b_i}\right); \quad i = 1;\ 2. \qquad (2)$$

On the diagram we have plotted points for both nozzles under the same conditions as in Fig. 3 and, consequently, there are twice as many points in Fig. 4. We note that the test values for the interface location contain a systematic error: the points for nozzle I lie generally below curve 2 and the points for nozzle II lie mostly above curve 2 in Fig. 4. As a consequence, the dispersion of test points about curve 2 is equal to 0.04 here.

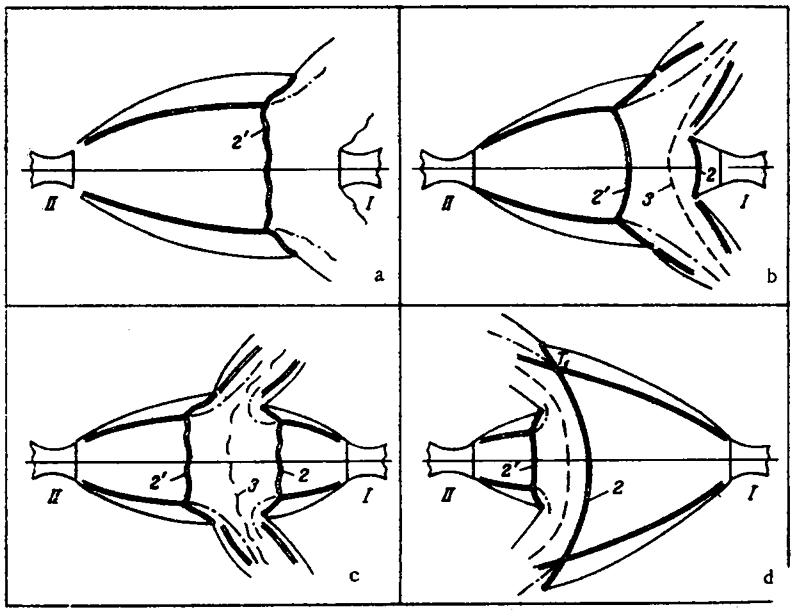

Fig. 2. Changes in the structure of interacting jets under increasing pressure.

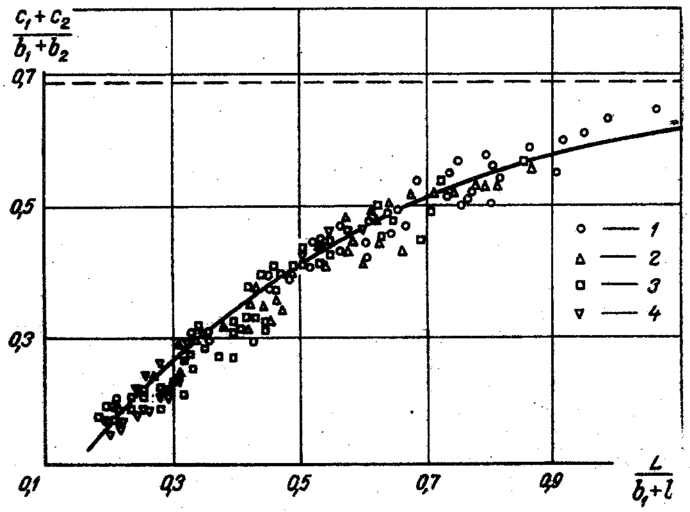

Fig. 3. Sum of the distances from the central density jumps, as a function of the discharge parameters: 1) $M_1 = M_2 = 1.0$; $d_1 = d_2 = 12$ mm; 2) $M_1 = 2.0$; $M_2 = 1.0$; $d_1 = d_2 = 12$ mm; 3) $M_1 = 1.0$; $M_2 = 2.0$; $d_1 = d_2 = 12$ mm; 4) $M_1 = 1.0$; $d_1 = 12$ mm; $M_2 = 2.0$; $d_2 = 15$ mm; solid line represents relation (1), dashed line represents the location of the Mach circle in the first "roll" of a free jet.

Formulas (1) and (2) together with the stipulation that $s_1 + s_2 = L$ constitute a system of equations from which the four unknowns c_1, c_2, s_1, and s_2 can be determined. Owing to the approximate character of these equations and to the dispersions associated with them, the calculated location of the central jumps in interacting jets may not necessarily correspond to the obvious physical requirement that the stagnation pressures at point Q on the interface be equal (Fig. 1):

$$P'_{\mathrm{I}} = P'_{\mathrm{II}}. \tag{3}$$

For determining the location of the central jumps and of the interface more rigorously, we propose a calculation method where c_1 and c_2 are determined from condition (3) at the stagnation point Q rather than from Eq. (2). This condition relates the values of the Mach number M_i' before the central jumps in the respective jets.

Since $P_{\mathrm{I}}\sigma_{\mathrm{I}} = P_{\mathrm{II}}\sigma_{\mathrm{II}}$, hence

$$\left[\frac{2}{k+1}\left(\frac{1}{M'_{\mathrm{I}}}\right)^2 + \frac{k-1}{k+1}\right]^{-\frac{k}{k-1}}\left[\frac{2k}{k+1}(M'_{\mathrm{I}})^2 + \frac{k-1}{k+1}\right]^{-\frac{1}{k-1}}$$

$$= \frac{P_{\mathrm{II}}}{P_{\mathrm{I}}}\left[\frac{2}{k+1}\left(\frac{1}{M'_{\mathrm{II}}}\right)^2 + \frac{k-1}{k+1}\right]^{-\frac{k}{k-1}}\left[\frac{2k}{k+1}(M'_{\mathrm{II}})^2 - \frac{k-1}{k+1}\right]^{-\frac{1}{k-1}}. \tag{4}$$

The profile of the Mach number $M_i(x)$ along the axis of a supersonic jet is accurately enough described by the approximate formula [6]

$$M(x) = M_{\mathrm{w}} + A\,\frac{(x - x_{\mathrm{w}})^k}{x}, \tag{5}$$

where $A = 2.35 + 4.5\,(0.426\,M - 1)\,(0.834\,k - 1)$; x_W is the space coordinate and M_W is the Mach number at the point where the first characteristic of an expansion wave intersects the jet axis; the latter is determined from the relation

$$\omega(M_{\mathrm{w}}) = \omega(M) + 2\theta.$$

Then

$$x_{\mathrm{w}} \simeq \sqrt{M_{\mathrm{w}}^2 - 1}.$$

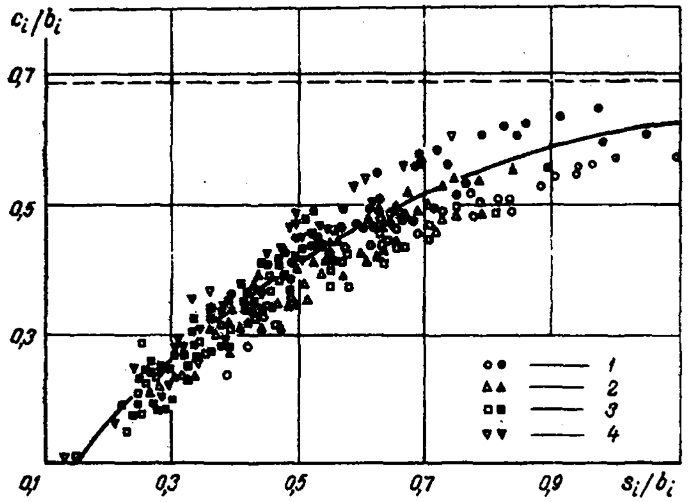

Fig. 4. Location of the central density jump, as a function of the interface location, in an individual jet. Notation is the same as in Fig. 3 (solid line represents relation (2)) black marks refer to nozzle II.

The distance x in formula (5) must be referred to the nozzle radius. This formula yields the location of a central jump in a jet, if the Mach number before that jump M_i':

$$M_i' = \left[M_w + A\,\frac{(c - x_w)^k}{c}\right]_i \tag{6}$$

is known.

The sum of the values obtained for c_1 and c_2 must, at the same time, satisfy Eq. (1). Thus, the system of Eqs. (1), (4), (6) yields the values of c_1 and c_2. The location of the interface, defined by s_1 and s_2, is then determined from Eq. (2) with the values of c_1 and c_2 already known. Its distance from nozzle I is, finally,

$$s = \frac{s_1 + L - s_2}{2}. \tag{7}$$

When calculations are made according to Eq. (7), it is possible to reduce the effect of the systematic error inherent in Eq. (2). System (1), (4), (6) is solved numerically or graphically with the aid of the axial pressure profile $P_i'(x)$ common to both jets, also with the aid of the curves in Figs. 3 and 4.

With the proposed empirical formulas (1) and (2), in combination with the physical requirement (3) of equal stagnation pressures at point Q on the interface, it is possible to calculate the location of the central jumps and of the interface between opposing coaxial jets within the interaction zone of their first "rolls."

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NOTATION

M	is the Mach number at the nozzle throat;
θ	is the half the divergence angle of a nozzle;
P	is the pressure in an isentropically stagnated stream, or the pressure in a receiver;
P'	is the total pressure behind a straight density jump;
M'	is the Mach number at the jet axis before a central density jump;
p_n	is the pressure at the nozzle throat;
p_a	is the ambient pressure;
$n = p_n/p_a$	is the discharge inefficiency;
k	is the adiabatic exponent for air;
$\omega(M)$	is the Prandtl–Mayer function;
$\sigma(M) = P'/P$	is the drop in total pressure at a straight jump;

d is the nozzle diameter at the throat;
L is the distance between nozzles from throat to throat;
c is the distance from a central density jump;
s is the distance from the interface;
x is the axial coordinate. All linear dimensions are referred to the diameter of the respective nozzle.

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